**eTruth tables**

**Exercise 1**

Check the following properties for (’*nor*’), (’*nand*’) and (’*xor*’) connectives using the truth table method.



1. and



**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ ⊕**

**Meta-symbols (express binary semantic relations):**

**|=**

**≡ (logical equivalence relation)**

**Theoretical result:**

**U ≡ V iff (if and only if) they have the same truth table.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **p** | **q** | ***r*** | **q v r** | **U=p ↑ (q v r)** | **p ↑q** | **p ↑ r** | **V=(p ↑q) ^ (p ↑r)** |  |  |  |  |
| **i1** | T | T | T | T | F | F | F | F |  |  |  |  |
| **i2** | T | T | F | T | F | F | T | F |  |  |  |  |
| **i3** | T | F | T | T | F | T | F | F |  |  |  |  |
| **i4** | T | F | F | F | T | T | T | T |  |  |  |  |
| **i5** | F | T | T | T | T | T | T | T |  |  |  |  |
| **i6** | F | T | F | T | T | T | T | T |  |  |  |  |
| **i7** | F | F | T | T | T | T | T | T |  |  |  |  |
| **i8** | F | F | F | F | T | T | T | T |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
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**Conclusion: U and V have identical truth tables, so they are logically equivalent. The second logical equivalence is the dual logical equivalence of the first one, obtained by interchanging dual connectives.**

**Exercise 2**

Using the truth table method decide what kind of formula (consistent, inconsistent, tautology, contingent) is . Write all the models and anti-models of .



|  |
| --- |
| Definitions:   1. A model of U is an interpretation which evaluates the formula U as true. 2. An anti-model of U is an interpretation which evaluates the formula U as false. 3. U is consistent if it has at least one model. 4. U is inconsistent if it has no models, all its interpretations are anti-models 5. U is a tautology (valid formula) if it is evaluated as true in all its interpretations. 6. U is a contingent formula if it is consistent, but not valid (it has at least one model and one anti-model) |
| Fp – the set of propositional formulas  S\_consistent – the set of consistent formulas  S\_contingent – the set of contingent formulas  S\_valid – the set of valid formulas (tautologies)  S\_inconsistent – the set of inconsistent formulas  Fp= S\_consistent U S\_inconsistent  S\_consistent = S\_valid U S\_contingent |

**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ ⊕**

**Meta-symbols (express binary semantic relations): |= ≡**

**U7 = p → ( p ˄ r ) V q**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ii | **p** | **q** | ***r*** | **( p ˄ r )** | **( p ˄ r ) V q** | **U7** |  |  |  |  |  |  |
| **i1** | T | T | T | T | T | T |  |  |  |  |  |  |
| **i2** | T | T | F | F | T | T |  |  |  |  |  |  |
| **i3** | T | F | T | T | T | T |  |  |  |  |  |  |
| **i4** | T | F | F | F | F | F |  |  |  |  |  |  |
| **i5** | F | T | T | F | T | T |  |  |  |  |  |  |
| **i6** | F | T | F | F | T | T |  |  |  |  |  |  |
| **i7** | F | F | T | F | F | T |  |  |  |  |  |  |
| **i8** | F | F | F | F | F | T |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
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**Conclusion: U7 is a contingent formula, having 7 models and 1 anti-model**

**Models: i1, i2,i3, i5, i6, i7, i8 :{p,q,r}->{T,F}**

i1(p)= T ,i1(q)= T ,i1(r)= T , i1(U7)=T

i2(p)= T ,i2(q)= T ,i2(r)= F , i2(U7)=T

i3(p)= T ,i3(q)= F ,i3(r)= T , i3(U7)=T

i5(p)= F ,i5(q)= T ,i5(r)= T , i5(U7)=T

i6(p)= F ,i6(q)= T ,i6(r)= F , i6(U7)=T

i7(p)= F ,i6(q)= F ,i6(r)= T , i6(U7)=T

i8(p)= F ,i8(q)= F ,i8(r)= F , i8(U7)=T

## **Anti-models: i4 : {p,q,r} -> {T,F}**

i4(p)= T ,i4(q)= F ,i4(r)= F , i4(U7)=F

**Exercise 3**

Using the truth table method, check whether the following logical consequences hold:

7.



**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ ⊕**

**Meta-symbols (express binary semantic relations): |= ≡**

**Theoretical result:**

**U |= V (V is a logical consequence of U) iff every model of U is also a model of V**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **p** | **q** | ***r*** | **U=p->q** | **q->r** | **q v r** | **p->q v r** | **V=(q->r)->(p->q v r)** |  |  |  |  |
| **i1** | T | T | T | T | T | T | T | T |  |  |  |  |
| **i2** | T | T | F | T | F | T | T | T |  |  |  |  |
| **i3** | T | F | T | F | T | T | T | T |  |  |  |  |
| **i4** | T | F | F | F | T | F | F | F |  |  |  |  |
| **i5** | F | T | T | T | T | T | T | T |  |  |  |  |
| **i6** | F | T | F | T | F | T | T | T |  |  |  |  |
| **i7** | F | F | T | T | T | T | T | T |  |  |  |  |
| **i8** | F | F | F | T | T | F | T | T |  |  |  |  |
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**I1, i2, i5, i6, i7, i8 – models of U**

**Conclusion : i1,i2,i5,i6,i7,i8 are models for U and also models of V, so V is a logical consequence of U**

**U≡V iff U |= V and V |= U**

**Exercise 4.**

Prove that the following formulas are tautologies using the truth table method.

1. the left-distribution of ’’ over ’’:



**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ ⊕**

**Meta-symbols (express binary semantic relations): |= ≡**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **p** | **q** | ***r*** | **q ↔r** | **U= p   → (q ↔r)** | **p → q** | **p → r** | **V = (p → q) ↔ (p → r)** | **U →V** |  |  |  |
|  | T | T | T | T | T | T | T | T | T |  |  |  |
|  | T | T | F | F | F | T | F | F | T |  |  |  |
|  | T | F | T | F | F | F | T | F | T |  |  |  |
|  | T | F | F | T | T | F | F | T | T |  |  |  |
|  | F | T | T | T | T | T | T | T | T |  |  |  |
|  | F | T | F | F | T | T | T | T | T |  |  |  |
|  | F | F | T | F | T | T | T | T | T |  |  |  |
|  | F | F | F | T | T | T | T | T | T |  |  |  |
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**Conclusion: the formula is a tautology because it is evaluated as true in all interpretations.**

**2**

**Normal forms**

1. A formula is in *disjunctive normal form* (DNF), if it is written as a disjunction of cubes:

, where are literals.



1. A formula is in *conjunctive normal form* (CNF), if it is written as a conjunction of clauses:

, where are literals.



**Exercise 5**

Transform the formulas into their equivalent conjunctive and disjunctive normal forms. Using one of these forms prove that are valid formulas in propositional logic.



**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ ⊕**

**Meta-symbols (express binary semantic relations): |= ≡**

**Theoretical results**.

1. To obtain DNF/CNF we apply the normalisation algorithm

**Exercise 6**

Using the appropriate normal form write all the models of the following formulas:

**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ ⊕**

**Meta-symbols (express binary semantic relations): |= ≡**



**U7=(qVr →p) →(p →r) ˄ q , replace →, using X→Y ≡Ꞁ X v Y**

**≡ Ꞁ (qvr →p) v (p →r) ˄ q , replace →**

**≡ Ꞁ (Ꞁ (qvr) v p) v (Ꞁ p v r) ˄ q**

**≡ ((q v r ) ˄ Ꞁp) v (Ꞁ p v r) ˄ q, apply distributive laws for the 2 subformulas**

**≡ ~~(q ˄ Ꞁp)~~ v (r ˄ Ꞁp) v (Ꞁp ˄ q) v (r ˄ q) DNF with 3 cubes ; U v U = U by the idempotency**

**The cubes provide the models of the formula.**

**Cube: Ꞁp ˄ q provides the models:**

**i1,i2:{p,q,r) -> {T,F}**

**i1(p) = F ; i1(q) = T ; i1(r) = F**

**i2(p) = F ; i2(q) = T ; i2(r) = T**

**Cube: r ˄ Ꞁp provides the models:**

**i3,i4:{p,q,r) -> {T,F}**

**i3(p) = F ; i3(q) = F ; i3(r) = T**

**i4(p) = F ; i4(q) = T ; i4(r) = T**

**Cube : r ˄ q**

**i5,i6:{p,q,r) -> {T,F}**

**i5(p) = F ; i5(q) = T ; i5(r) = T**

**i6(p) = F ; i6(q) = T ; i6(r) = T**

**We notice that i2 = i4 = i5.**

**The models of U are i1, i2, i3, i6 (U7)=T**

**i1(U7) = T …....**

**Exercise 7**

Using the appropriate normal form, prove that the following formulas are inconsistent:



**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ ⊕**

**Meta-symbols (express binary semantic relations): |= ≡**

**Exercise 8**

Write all the anti-models of the following formulas using CNF.



**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ ⊕**

**Meta-symbols (express binary semantic relations): |= ≡**

DNF(U7)=(**Ꞁ** p **˄** not(r)